Department of Geodetic Science

DATA ANALYSIS IN CONNECTION WITH THE NATIONAL GEODETIC SATELLITE PROGRAM (II)

Second Semiannual Status Report

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PREFACE

This project is under the supervision of Ivan I. Mueller, Professor of the Department of Geodetic Science at The Ohio State University, and it is under the technical direction of Jerome D. Rosenberg, Project Manager, Geodetic Satellites Program, NASA Headquarters, Washington, D. C. The contract is administered by the Office of University Affairs, NASA, Washington, D. C. 20546.

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1. STATEMENT OF WORK

The primary objective of the OSU investigation is the geometric analysis of geodetic satellite data. In order to fulfill this objective the following tasks are being attempted during the current grant period.

(1) Extension of the computer programs which can handle optical and/or range observations for range and range rate, Doppler, and laser systems.

(2) Development of data preprocessing systems to handle the data from the above observation methods.

(3) Analysis of observational data for the National Geodetic Satellite Program for the purpose of fulfilling the primary objective as observational data becomes available.

(4) The development of computer programs for the adjustment of optical, ranging and Doppler data in the short-arc mode.

(5) Theoretical investigations into sequential least squares adjustment of simultaneous satellite observations in combination with terrestrial data.

2. ACCOMPLISHMENTS DURING THE REPORT PERIOD

2.1 Data Preprocessing

2.11 Preprocessing Electronic Observations

The investigation of the preprocessing procedures of electronic satellite observations is finished; the results were submitted in a separate report in March (OSU Department of Geodetic Science Report No. 100).

2.12 Preprocessing Optical Observations

During the report period all available simultaneous MOTS and PC-1000 data was selected and requested from the Data Center. Due to the fact that the computer programs at OSU require a different card format, as well as a different station numbering system, several computer programs had to be written to renumber, reformat and sequence all of this data. There were approximately 7000 observations selected. The time system required for the OSU computer programs is UT1. The time given for each PC-1000 was satellite clock time, and the correction from this clock time to UT2 had to be interpolated from the APL graph. It was necessary to hand correct every PC-1000 observation to get the time into the UTC system. The MOTS data was given in the UTC system, but the last digit of this time (tenths of milliseconds) was rounded off. It was necessary to examine every MOTS card and correct for this roundoff. The time correction from UTC to UT1 was performed by a computer program using the second difference Bessel formula to interpolate from tables given by the US Naval Observatory. All data is now in the UT1 time system. The correction for parallactic refraction was one of the most difficult and required two different computer programs. The first step was to compute the range and local apparent sidereal time for each data point and write a tape, and the second program takes this tape and computes the parallactic refraction and applies it to the right ascension and declination.

On the utilization of the BC-4 data received from the Data Center, see section 2.4.

2.2 Investigations into the Economics of Plate Reduction Procedures

Extensive work was undertaken to find answers to the following questions:

1. What is the loss of accuracy when multiple satellite directions are determined

   (a) using the whole star field on a plate and photogrammetric reduction techniques,

   (b) using partial star fields around the individual satellite images and astrometric reduction techniques.

2. How do the precisions of station coordinates compare when the satellite image coordinates on (BC-4) plates are utilized in one of the following manners:

(a) using one fictitious image per plate obtained by fitting fifth-degree polynomials through the individual satellite coordinates,

(b) using three fictitious images per plate obtained by fitting third-degree polynomials to three separate sections of the satellite trail, each consisting of 60-90 images.

(c) using 3, 6, 10, and 20 individual images per plate without attempting to fit any polynomial.

Partial answers to these questions were submitted in separate reports in March and in June (OSU Department of Geodetic Science Reports No. 106 and No. 110). See also section 2.4 of this report.

2.3 The Durham Investigation on the Compatibility of Plate Reduction Methods of the Participating Agencies

The background for the problem is as follows. In May, 1967, the NGSP Working Committee on the Statistical Combination of Satellite Observational Data, meeting in Durham, North Carolina, recommended that photographic plates taken by different agencies (National Aeronautics and Space Administration, NASA; Aeronautical Chart and Information Center, United States Air Force, ACIC; Smithsonian Astrophysical Observatory, SAO; and United States Coast and Geodetic Survey, Environmental Science Services Administration, ESSA) be interchanged and reduced according to each agency's normal procedure, the results being in the form normally sent to the Geodetic Satellite Data Center. Although plates were not actually interchanged, ESSA supplied data from three photographic plates in the form of measured star and satellite coordinates. The three plates were exposed simultaneously at the following stations:

Plate No. 2559	Lynn Lake, Manitoba
Plate No. 6132	Frobisher Bay, NWT
Plate No. 5205	Cambridge, NWT

The event occurred on November 30, 1965, the satellite being Echo II. Twenty-one satellite images were selected on each plate as test points for which directions in space were to be determined by each agency.

In October, 1967, OSU forwarded to each agency pertinent data including punched cards containing the measured star and satellite coordinates, the stars' SAO catalog positions, and other necessary information such as timing data, atmospheric conditions and other items that will be discussed in more detail in the next section of this report. The agencies were requested to use the given data in their own reduction programs and to forward the results to the Department of Geodetic Science in the same format as used for the Geodetic Satellite Data Center.

2.31 General

The agency-computed directions for the 21 satellite images on each plate are given in Tables 1, 2, and 3. These directions correspond to the apparent place of a star. That is, considering the satellite images as those of unknown stars, the right ascensions and declinations listed correspond to those of the reference stars updated for precession, nutation, proper motion, and annual aberration from catalog epoch to observation epoch. They are not corrected for effects of astronomic refraction or diurnal aberration, for phase angle and parallactic refraction.

Table 4 contains a reduction procedure summary for the four agencies. The procedures given thereon were documented before the agencies performed the reductions that are the subject of this report. Table 5 shows the agencies' actual procedures used for this problem as described to OSU in letters of transmittal of final results and during personal conversations with the respective agencies. This matter will be discussed in more detail in subsequent paragraphs. It is obvious from the tables that all data was not received with the same corrections having been made nor was this requested. The satellite positions returned were to be in the same form as normally sent to the GSDC with the agencies' normal preprocessing procedures having been followed. Therefore, it was necessary for OSU to perform a "homogenization" of the data so that a comparison could be made. As stated it was decided to compare the agency directions in the same

Table 1

Plate 2559

Agency Computed Directions for 21 Satellite Images after Corrections to Obtain the Apparent Place

	RA						DEC					
No.			ESSA	ACIC	SAO	NASA			ESSA	ACIC	SAO	NASA
	h	m	s	s	s	s	°	'	"	"	"	"
126	09	39	53.749	53.698		54.038	59	59	49.92	50.53		51.32
143		35	37.350	37.252		37.611	60	08	27.62	28.38		28.92
161		30	59.715	59.581		59.947		17	03.13	04.03		04.26
179		26	16.275	16.051		16.477		24	58.50	59.49		59.54
201		20	23.739	23.436		23.914		33	51.39	52.46		52.21
219		15	29.723	29.347		29.869		40	22.60	23.74		23.30
239		09	57.425	56.966	57.210	57.545		46	43.58	44.82	42.90	44.14
258		04	36.014	35.475	35.743	36.110		51	58.75	59.95	58.09	59.14
281	08	58	00.267	59.641	59.978	00.340		57	02.21	03.35	01.62	02.39
299		52	45.519	44.818	45.246	45.573	61	00	09.67	10.74	09.08	09.74
313		48	38.727	37.972	38.485	38.769		02	01.90	02.90	01.32	01.79
330		43	34.972	34.157	34.790	34.998		03	31.18	32.10	30.58	30.93
350		37	34.725	33.837	34.618	34.734		04	16.57	17.30	16.10	16.17
368		32	07.051	06.108	07.027	07.043		04	03.61	04.19	03.21	03.05
387		26	18.433	17.445	18.490	18.409		02	47.54	47.87	47.37	46.85
403		21	22.451	21.436	22.575	22.413		00	50.28	50.45	50.28	49.43
414		17	58.256	57.220		58.210	60	59	03.74	03.71		02.76
434		11	45.086	44.028		45.024		54	54.15	53.88		53.03
452		06	08.506	07.444		08.426		50	05.75	05.24		04.45
473	07	59	34.768	33.731		34.669		43	11.79	10.98		10.29
494		53	00.299	59.300		00.179		34	52.31	51.21		50.65

Table 2

Plate 5205

Agency Computed Directions for 21 Satellite Images after Corrections to Obtain the Apparent Place

	RA						DEC					
No.	h	m	ESSA s	ACIC s	SAO s	NASA s	°	'	ESSA "	ACIC "	SAO "	NASA "
126	3	55	14.120	13.984		14.251	61	49	05.42	03.52		04.48
143		51	15.902	15.838		16.032	60	58	41.72	39.82		40.85
161		47	18.996	19.007		19.126	60	05	09.82	08.09		09.13
179		43	36.227	36.308		36.362	59	11	23.43	21.90		22.92
201		39	21.343	21.491		21.491	58	05	25.91	24.62		25.51
219		36	06.078	06.263		06.221	57	11	23.36	22.16		23.12
239		32	42.139	42.348		42.276	56	11	21.83	20.58		21.57
258		29	39.209	39.426	39.176	39.329	55	14	17.16	15.75	16.64	16.87
281		26	11.994	12.201	11.918	12.091	54	05	26.66	24.92	25.61	26.22
299		23	39.987	40.179	39.894	40.071	53	11	43.38	41.31	42.39	42.69
313		21	46.348	46.532	46.248	46.413	52	29	59.84	57.56	59.08	59.04
330		19	34.862	35.026	34.767	34.911	51	39	38.25	35.74	37.88	37.30
350		17	07.992	08.135	07.899	08.024	50	40	35.85	33.19	36.05	34.80
368		15	02.746	02.871	02.659	02.765	49	47	44.15	41.48	44.94	43.09
387		12	57.072	57.183		57.084	48	52	18.30	15.79		17.26
403		11	16.222	16.325		16.229	48	05	56.61	54.42		55.73
414		10	09.092	09.186		09.099	47	34	15.91	14.06		15.11
434		08	11.871	11.959		11.875	46	36	54.90	53.68		54.42
452		06	31.795	31.869		31.799	45	45	44.72	44.20		44.51
473		04	40.072	40.138		40.074	44	46	35.67	35.99		35.84
494		02	53.874	53.919		53.874	43	48	02.54	03.62		02.99

Table 3

Plate 6132

Agency Computed Directions for 21 Satellite Images after Corrections to Obtain the Apparent Place

No.	h	m	ESSA s	ACIC s	SAO s	NASA s	°	'	ESSA "	ACIC "	SAO "	NASA "
126	17	51	11.346	10.685		11.133	54	22	51.19	48.30		51.52
143		55	35.996	35.296		35.779		31	30.39	27.52		30.61
161	18	00	21.046	20.302		20.827		39	59.90	57.07		00.01
179		05	12.138	11.365		11.918		47	48.56	45.79		48.61
201		11	15.366	14.562		15.151		56	24.21	21.47		24.13
219		16	18.359	17.531		18.152	55	02	33.02	30.37		32.87
239		22	01.026	00.177	01.032	00.832		08	26.59	24.02	25.93	26.34
258		27	32.612	31.753	32.661	32.438		13	03.89	01.53	03.34	03.59
281		34	21.675	20.804	21.743	21.528		17	19.83	17.66	19.44	19.42
299		39	47.100	46.233	47.157	46.971		19	35.88	33.84	35.74	35.40
313		44	02.443	01.590	02.488	02.337		20	42.03	40.12	42.01	41.52
330		49	16.503	15.655	16.521	16.419		21	15.85	14.12	15.98	15.27
350		55	30.272	29.450	30.268	30.214		20	38.13	36.59	38.58	37.45
368	19	01	10.260	09.473	10.240	10.222		19	01.31	59.98	02.06	00.54
387		07	12.204	11.459	12.189	12.184		16	03.39	02.25	04.47	02.55
403		12	19.838	19.132		19.829		12	35.88	34.96		34.98
414		15	52.397	51.719		52.391		09	40.49	39.63		39.54
434		22	19.525	18.914		19.517		03	13.98	13.34		12.93
452		28	10.860	10.319		10.848	54	56	14.34	13.81		13.25
473		35	02.196	01.747		02.174		46	30.63	30.30		29.48
494		41	54.037	53.691		53.995		34	57.42	57.19		56.22

Table 4

Star Updating and Satellite Image Corrections by the Four Agencies for an Active Satellite from [Mueller, et al., 1967] and [Hotter, 1967]

	ACIC		ESSA		NASA		SAO	
	Star	Sat.	Star	Sat.	Star	Sat.	Star	Sat.
Catalogue	BOSS to SAO		SAO		SAO		SAO	
Proper Motion	C		M		C		C	
Precession	C		M,C		M			
Nutation	C		C		C			
Annual Aber.	C		C		C			C
Diurnal Aber.	C		C		C			
Astro. Refr.	CP	-CP (w/adj. coef.)	CP	-CP	CP	-CP (w/adj. coef.)	Implicit in plate reduction	
Parall. Refr.						C		

M: matrix correction
C: conventional correction
CP: conventional during plate processing

Table 5

Star Updating and Satellite Image Corrections by the Four Agencies for this Event

	ACIC		ESSA		NASA		SAO	
	Star	Sat.	Star	Sat.	Star	Sat.	Star	Sat.
Catalogue	BOSS		SAO		SAO		SAO	
Proper Motion	C		M		C		C	
Precession	C		M,C		M			A
Nutation	C		C		C			A
Annual Aber.	C		C		C			C
Diurnal Aber.			C	-C	C	-A		
Astro. Refr.	CP	-CP (w/ adj. coef.)	CP	-CP	CP	-CP (w/adj. coef.)	Implicit in plate reduction	
Parall. Refr.		C (-A)				C (-A)		

A: added by OSU to make satellite directions compatible system as the apparent place of a star. Any other system of comparison would not fully credit the unique agency reduction since, for example, the SAO does not explicitly correct for diurnal aberration or parallactic refraction for their final satellite directions. It was a better procedure to remove a correction that was made explicitly to the star or satellite direction (knowing the agency formula) than to add a correction never explicitly or implicitly made by employing another agency's formula and coefficients. This procedure was violated in one case out of necessity.

2.32 The ESSA Reduction

The ESSA positions are the ones that have been used as the standard in the OSU astrometric investigations (updated from the positions given in Tables 1, 2, and 3 to the OSU "fictitious observed" system). ESSA performed a complete reduction for this problem as they normally do; that is, the parameters of interior orientation of each camera were adjusted along with the elements of exterior orientation instead of using coefficients from a previous camera calibration. It is re-emphasized that fixed parameters were used in the expression for astronomic refraction instead of adjustable ones. The same parameters were, therefore, used in updating the stars to the "observed" place as in removing astronomic refraction from the interpolated satellite directions. As stated, the correction for diurnal aberration was also removed from the interpolated satellite positions. Therefore, OSU did not make any corrections to the ESSA data. The standard errors of unit weight are repeated: Plate 2559, 2.96 μm; Plate 5205, 3.17 μm; and Plate 6132, 2.80 μm.

2.33 The ACIC Reduction

The ACIC made some adjustments to the furnished data. They shifted the origin of the plate coordinate system so that all positive coordinates resulted. The coordinates were converted to a plate negative coordinate system instead of using the contact positive coordinate system as given. The coordinate system was rotated

315° to compensate for the orientation of the plate when placed in the camera and the angle of camera rotation for the event. The number of stars and star images finally used per plate were as follows:

Plate 2559 — 51 stars and 166 star images

Plate 5205 — 59 stars and 217 star images

Plate 6132 — 56 stars and 169 star images

The ACIC deviated from their procedure documented in OSU Department of Geodetic Science Report No. 82. The adjustable parameters were the same as in Report No. 82 except that only the first two coefficients, η_1 and η_2, in the Garfinkel expression for astronomic refraction were adjusted. The others were constrained and chosen from tables. It must be noted that tabular values for all the Garfinkel coefficients had been employed when updating the stars. The reference stars were never corrected for diurnal aberration, and, therefore, this correction was not removed by OSU from the given satellite directions (nor had the ACIC removed it either for the same reason). Finally, the furnished satellite directions (two sets of directions were actually furnished, one with phase angle correction which was disregarded and one without—the one desired by OSU) had been corrected for parallactic refraction contrary to the procedure stated in Report No. 82.

OSU, therefore, had to remove the ACIC correction for parallactic refraction. This was done by using the formula used by ACIC and attributed to F. Hanson, the practiced version of which was received by conversation with the ACIC. This formula is given as

$$Z_{\text{corrected}} = Z_R + \Delta Z \left[1 - 29.275 \left(\frac{T}{H}\right)\right] \tag{1}$$

where 29.275 m/°C is the gas constant for air, T is the station temperature in degrees Kelvin, H is the height of the satellite in meters, Z_R is the refracted zenith distance, $Z_{\text{corrected}}$ is the zenith distance corrected for atmospheric refraction (astronomic refraction - parallactic refraction), and ΔZ equals the Garfinkel expression for astronomic refraction used by the ACIC. To correct the refracted zenith distance of the satellite for atmospheric refraction (which is equivalent to removing the effects of astronomic refraction and then correcting for parallactic refraction), the ACIC utilized in the expression for ΔZ the adjusted Garfinkel coefficients, η_1 and η_2, instead of the ones used when updating the stars.

For OSU to remove the correction for parallactic refraction it was necessary, therefore, to add the effects of atmospheric refraction, using equation (1), back into the given satellite directions to obtain Z_R and then to compute ΔZ to remove astronomic refraction. The OSU program for updating stars to the "observed" place with slight modifications was used. Since the expression for ΔZ employs the refracted zenith distance Z_R, it was necessary to iterate in the form

$$Z_R = Z_{\text{corrected}} - \Delta Z \left[1 - 29.275 \left(\frac{T}{H}\right)\right] \tag{2}$$

where $Z_{\text{corrected}}$ was given and Z_R desired. Successive values of Z_R were inserted in the expression for ΔZ in (2) until the differences in newly computed Z_R's were less than $0\overset{''}{.}01$. Then the final Z_R was used to computed ΔZ and remove astronomic refraction. That is, the unrefracted zenith distance Z_0 was computed by

$$Z_0 = Z_R + \Delta Z \tag{3}$$

where

$$\Delta Z = \eta_1 \tan\theta + \eta_2 \tan^3\theta + \eta_3 \tan^5 + \eta_4 \tan^7\theta \ , \tag{4}$$

θ is an auxiliary angle computed by

$$\cot 2\theta = \gamma_0 \cot Z_R , \tag{5}$$

and γ_0 is given by

$$\gamma_0 = 8.1578 \sqrt{273} / \sqrt{T} \tag{6}$$

where T is again the station temperature in degrees Kelvin. It must be reemphasized that OSU used the adjusted values of η_1 and η_2 in equation (4) in all computations. Table 6 gives the exact values of the necessary parameters as received from the ACIC. The temperature values given were not transmitted from the ACIC since they were already known by OSU.

The final right ascensions and declinations as given in Tables 1, 2, and 3, then, were equivalent to the apparent place of a star. In the original satellite positions sent by the ACIC, standard errors for each right ascension and declination were given, but it is not known how these were computed. In right ascension all the standard errors given were about 3".89 (Plate 2559), 2".50 (Plate 5205), and 4".12 (Plate 6132). In declination, they increased from 3".90 (image 126) to 5".14 (image 494) for Plate 2559, 3".19 (image 494) to 4".72 (image 126) for Plate 5205, and 6".56 (image 126) to 6".85 (images 387, 403, 414) and back down to 6".81 (image 494) for Plate 6132.

Table 6

ACIC Furnished Values of Parameters Used in Garfinkel Expression and Hanson's Formula to Remove Correction for Parallactic Refraction from ACIC and NASA Satellite Directions

	Plate 2559	Plate 5205	Plate 6132
η_1 *	$0.49495623 \times 10^{-2}$	$0.55217706 \times 10^{-2}$	$0.58339089 \times 10^{-2}$
η_2 *	$0.36110878 \times 10^{-2}$	$0.38490295 \times 10^{-2}$	$0.38318634 \times 10^{-2}$
η_3	$0.20751953 \times 10^{-2}$	$-0.19531250 \times 10^{-2}$	$0.12207031 \times 10^{-2}$
η_4	$0.45776367 \times 10^{-3}$	0.31250000×10^{0}	0.1562500×10^{-1}
H	1137 Km.	same	same
T **	-17.65 °C	-22.0 °C	-18.7 °C

* Adjusted
** Available from ESSA data furnished

2.34 The SAO Reduction

The SAO reduction was performed as stated in OSU Department of Geodetic Science Report No. 82 with necessary adjustments to allow for the fact that the usual Baker-Nunn film was not being reduced.

Since the SAO employs an astrometric reduction, the area reduced on each plate was restricted to keep nonlinear effects at a minimum, and, therefore, directions for all 21 images were not computed. Two images for each star were used in the reduction, the stars used being:

Plate 2559		Plate 5205		Plate 6132	
714	832	711	822	712	832
748	837	741	827	717	836
772	848	752	831	782	841
822	853	772	837	803	847
826	855	802	851	823	857
		813	855	826	

The SAO was faced also with the problem of multiple exposure times. Therefore, it was necessary to use fictitious right ascensions and declinations for the stars, computed in the epoch of observation, and then reduced back to the catalog epoch since the SAO updates its stars only for proper motion before reduction. It must be recalled that the model implicitly corrects for differential effects of aberration and refraction between images. The satellite positions, submitted in the 1950.0 epoch, without annual aberration added, were updated by OSU to the apparent place.

The SAO because of its astrometric reduction model used an iteration process to find the best origin of coordinates.

Residuals between measured and adjusted star image coordinates were given so that OSU was able to compute the standard errors of unit weight for the reductions. OSU assumed when computing the degrees of freedom (number of observations minus number of unknowns) that the number of unknowns was equal to 6. The standard errors of unit weight were: Plate 2559, 2.28 μm; Plate 5205, 2.67 μm; and Plate 6132, 2.61 μm.

2.35 The NASA Reduction

The NASA reduction was performed as stated in OSU Department of Geodetic Science Report No. 82, and the forwarded results were in the format expected. That is, they contained corrections for diurnal aberration and parallactic refraction. The calibration for the elements of interior orientation was performed by Duane Brown Associates. Normally, the camera calibration data is available and not adjusted for in each event. This data was also submitted along with the satellite directions.

To remove the corrections for diurnal aberration and parallactic refractiom, OSU used the same program as used for the ACIC data with the exception that a provision was made for removing diurnal aberration after the correction for parallactic refraction had been removed. The ACIC formulas and data as given in equations (1) to (6) and Table 6 were used to remove this correction although NASA applied it with different formulas. However, since the effect is small, the error committed in using the ACIC method and data was considered insignificant. The only change, then, in the program was to remove the correction for diurnal aberration from the satellite directions.

Standard errors in x and y measurements after adjustment were given per plate and are as follows: Plate 2559, 4.3 μm in x and 4.9 μm in y; Plate 5205, 3.9 μm in x and 3.8 μm in y; Plate 6132, 3.6 μm in x and 3.8 μm in y. NASA used 49 star images per plate in its reductions.

2.36 Comparison of the Agency Reductions

Figs. 1 through 6 show the deviations per satellite image of the ACIC, SAO, and NASA values given in Tables 1, 2, and 3 from the ESSA values given in those tables. That is, the ESSA minus Agency values have been plotted.

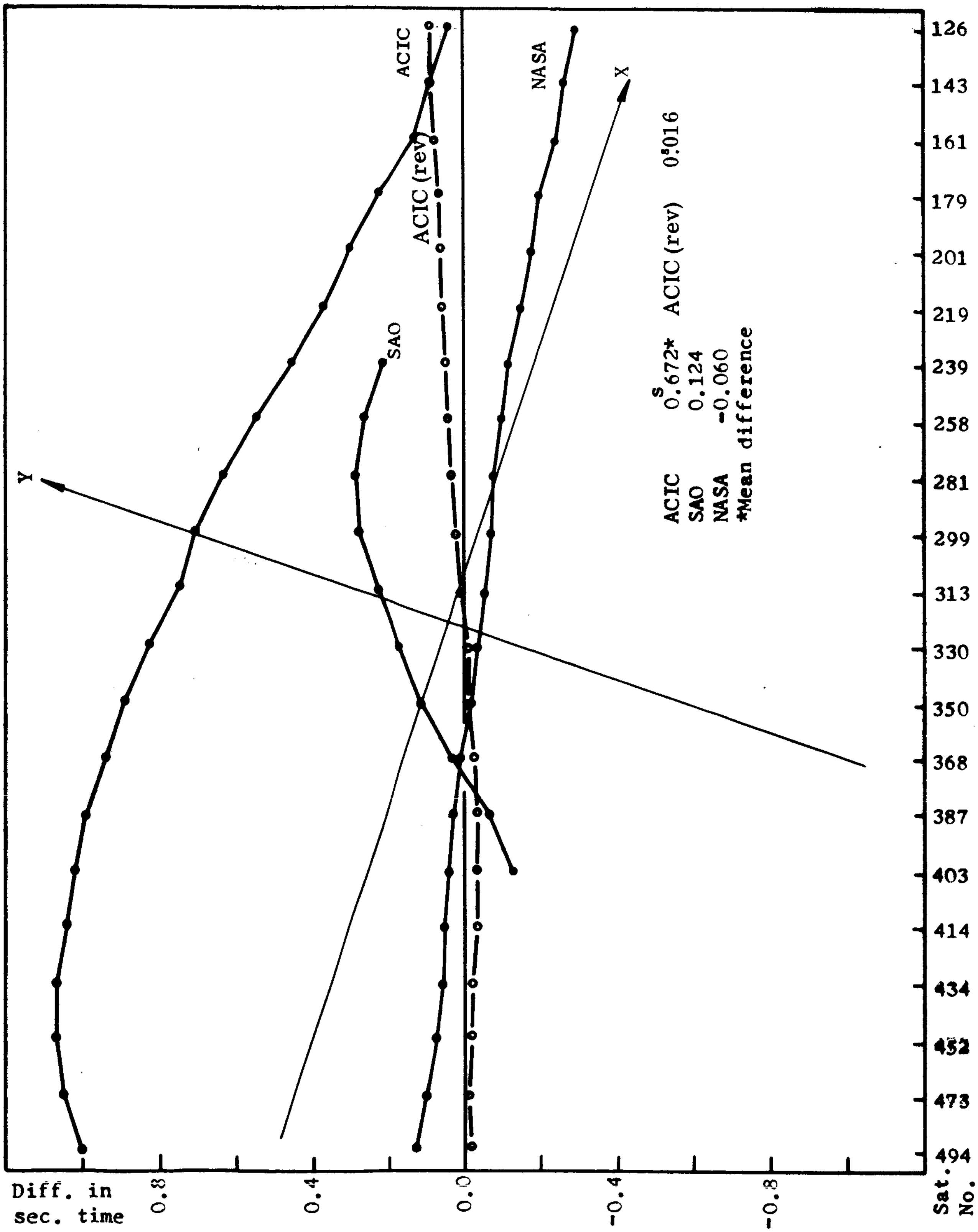

FIG. 1. PLATE 2559: ESSA RA - AGENCY RA

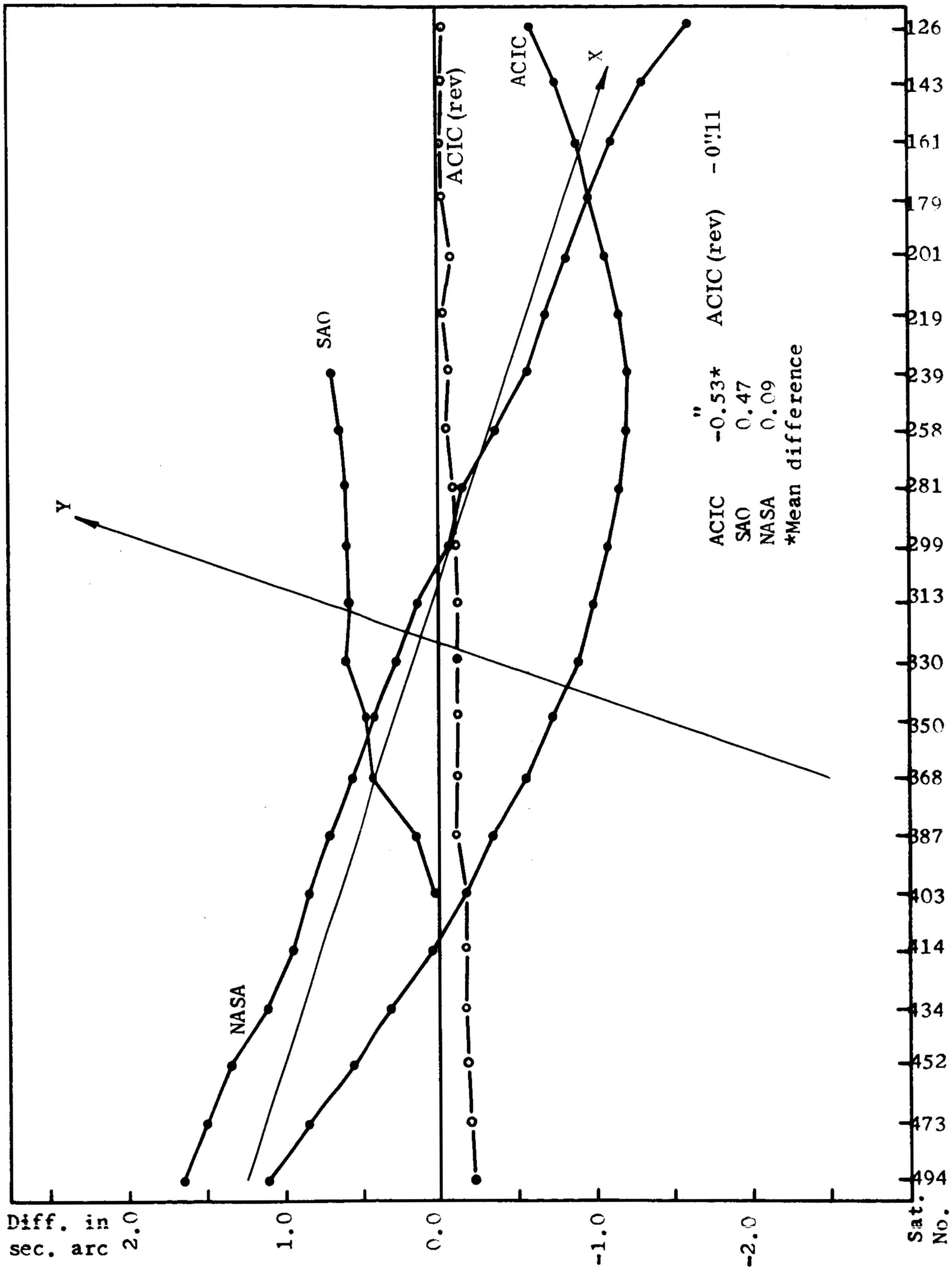

FIG. 2. PLATE 2559: ESSA DEC - AGENCY DEC

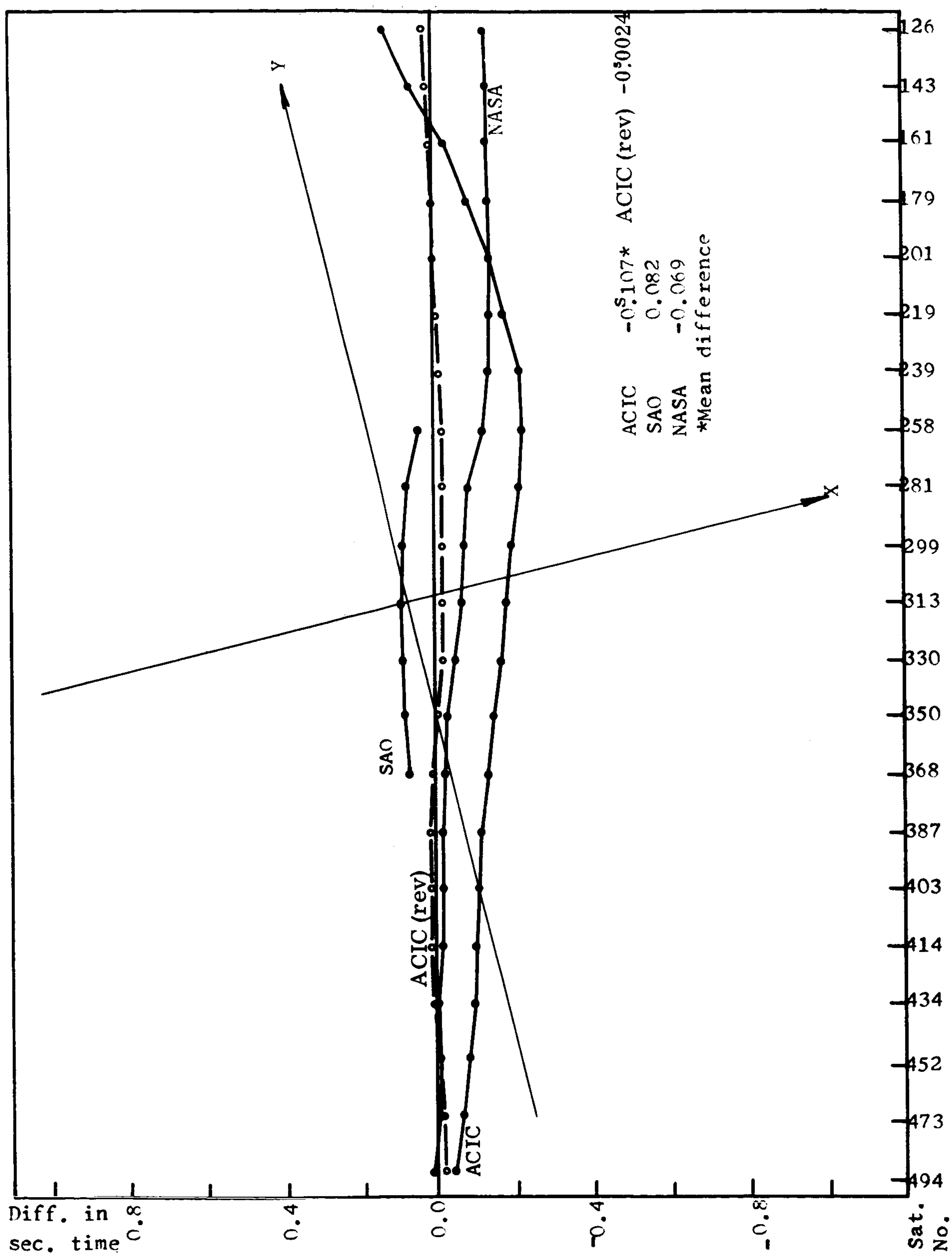

FIG. 3. PLATE 5205: ESSA RA - AGENCY RA

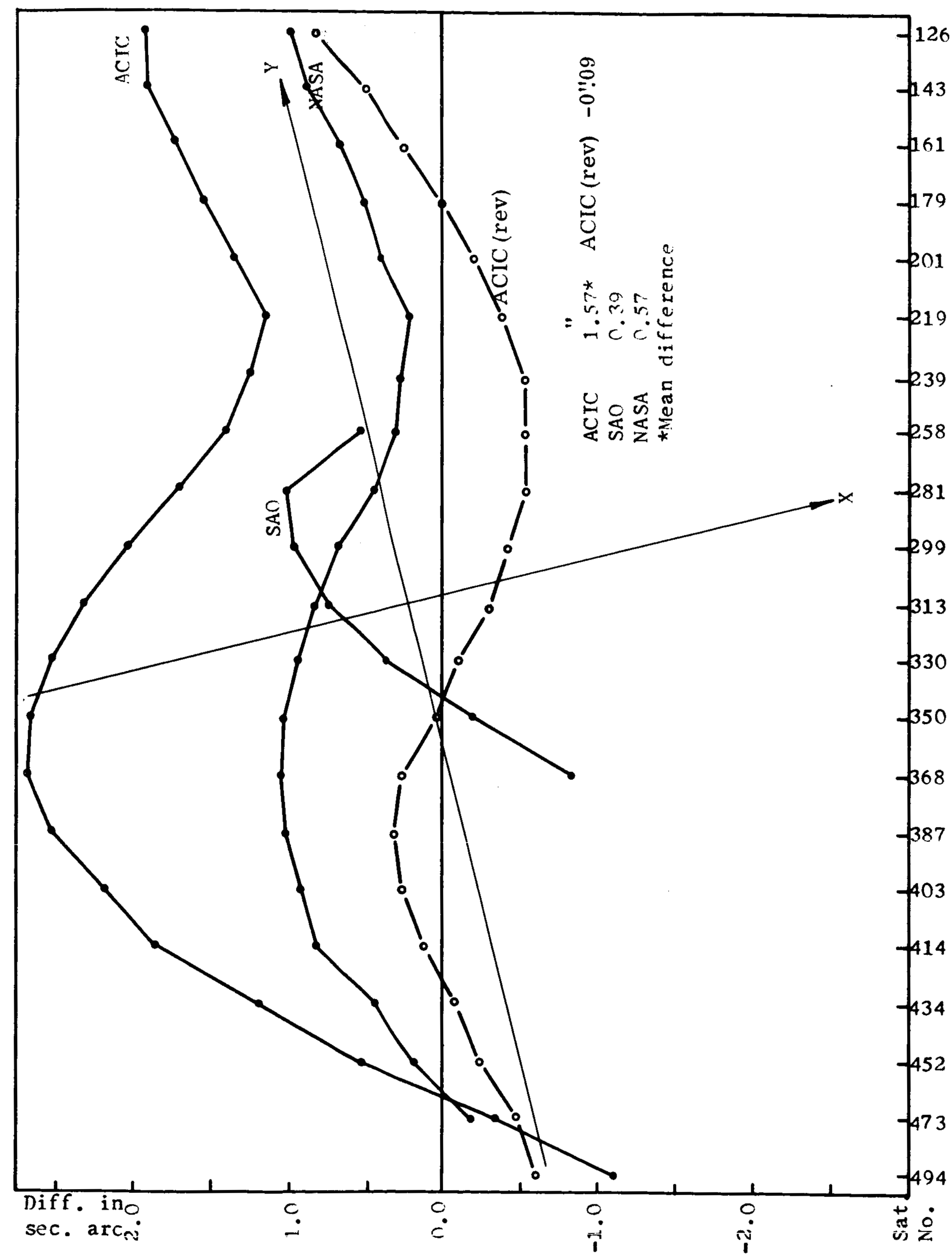

FIG. 4. PLATE 5205: ESSA DEC - AGENCY DEC

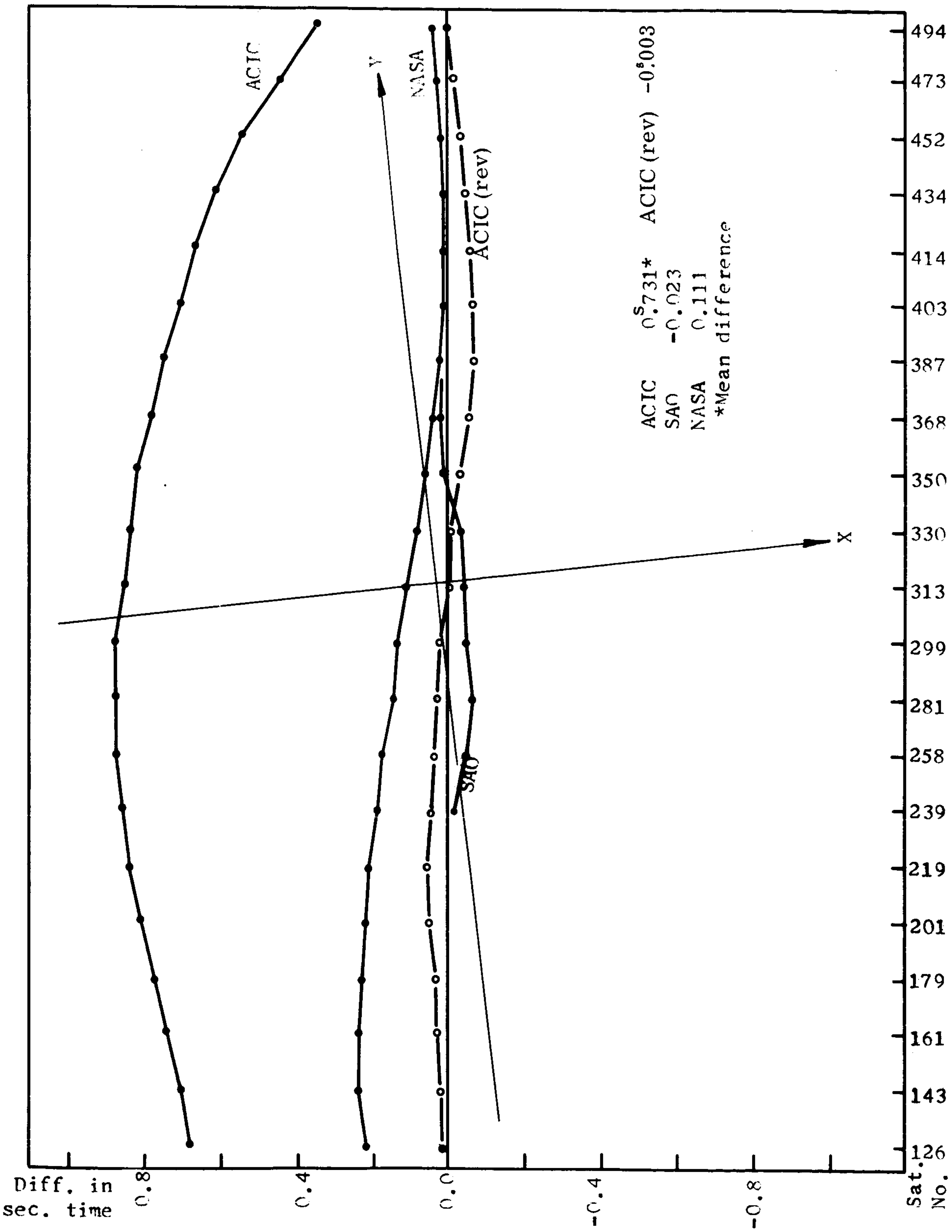

FIG. 5. PLATE 6132: ESSA RA - AGENCY RA

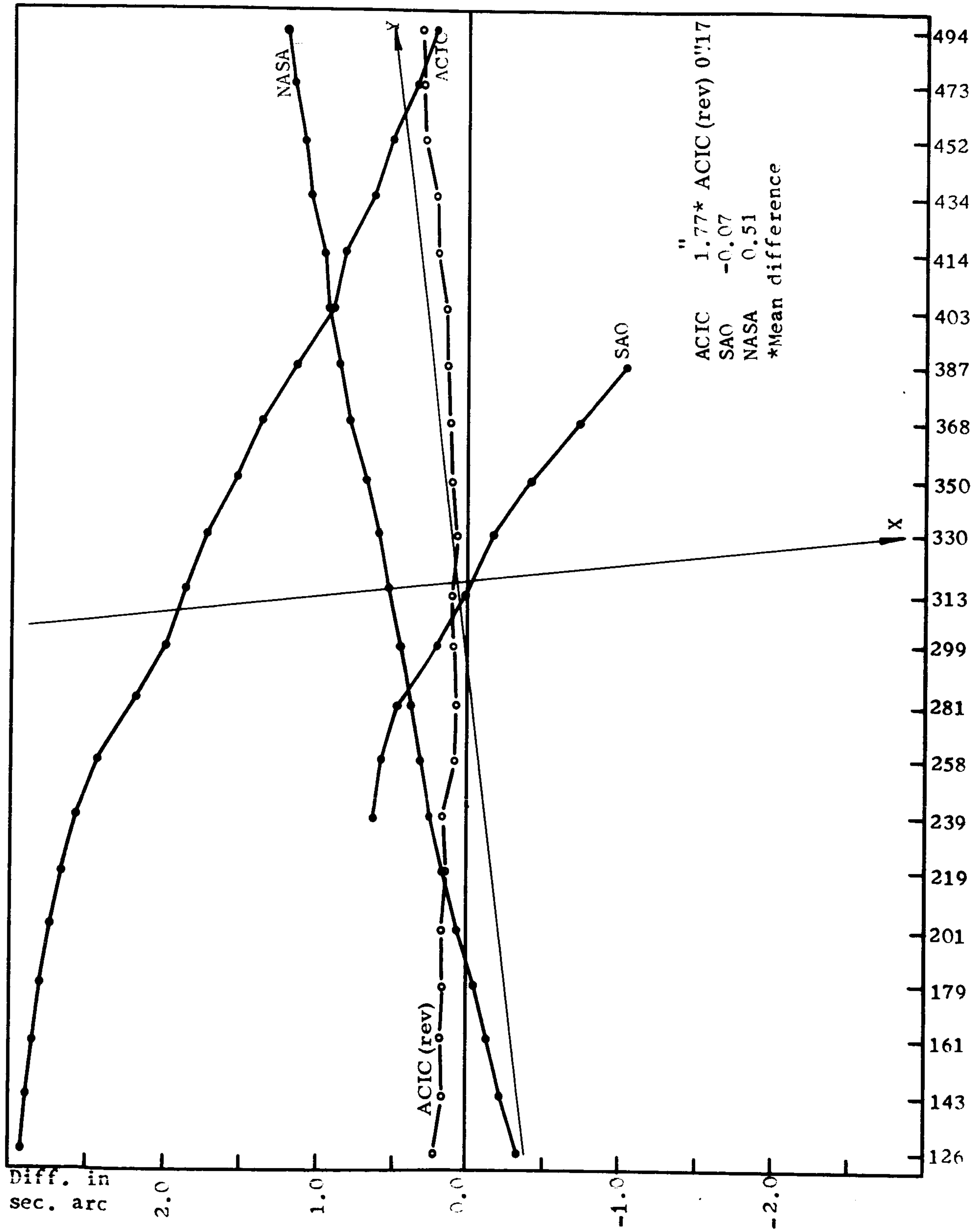

FIG. 6. PLATE 6132: ESSA DEC - AGENCY DEC

Figs. 7, 8, and 9 show the patterns of right ascension, declination, and zenith distance along the satellite trail. If the horizontal center line in each figure is regarded as the satellite trail, then the superimposed axes show the location of the plate center with respect to the trail. The figures also give the mean differences from the ESSA values. It must be understood that in this agency comparison the ESSA directions are not considered as the standard. The graphs show deviations from the ESSA values only because they had been used as the standard in the OSU investigations. Actually, any agency's could have been considered the reference values here.

The SAO results are remarkable considering that the stars are not updated except for motion before reduction and that their model must allow for differential effects of aberration and refraction. It is even more amazing since on two plates the SAO exceeded the OSU-computed radii (see OSU Department of Geodetic Science Report No. 106) of acceptable results for the "short" Turner's method. The fact that they used more star images and an iteration technique to find the origin of coordinates made a huge difference from the OSU reductions.

The good comparison of the SAO results and the fact that the differences from those of ESSA show symmetry about the plate center indicates that if satellite images occurred near the plate center and that if a curve-fitting procedure was going to be used to compute directions for a fictitious point at the plate center, then the astrometric model of the SAO would give equal results to those of the elaborate photogrammetric reductions.

The NASA results appear close to those of the SAO and ESSA in most cases with general symmetry about the plate center. Even though NASA uses a photogrammetric reduction, one does not expect exact comparison with the ESSA results since 49 reference stars were used as compared to about 120 for ESSA. For that matter, no two agencies will shown the same results although the average differences should be small.

The ACIC results were disturbing. They differed in some cases by

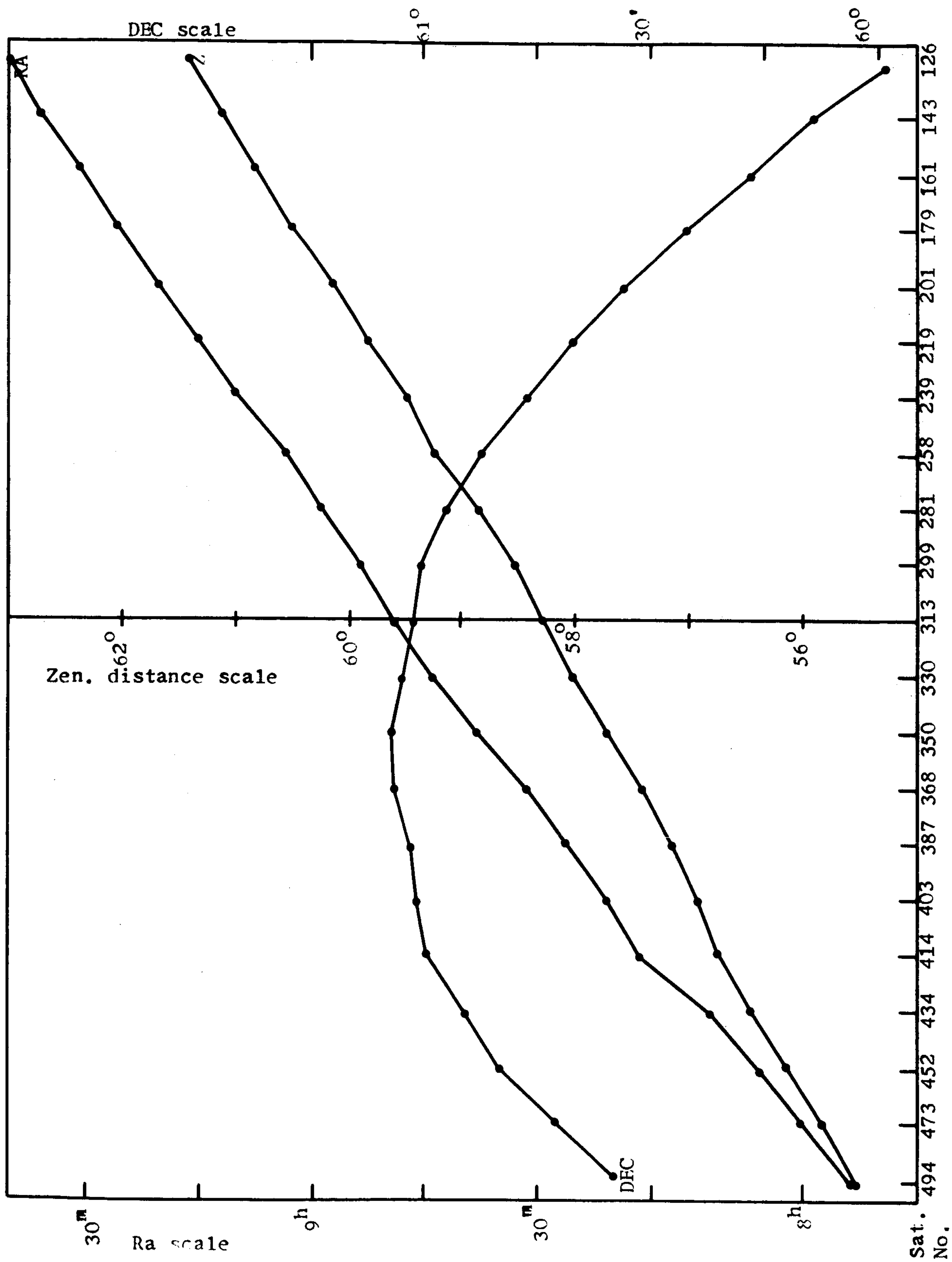

FIG. 7. PLATE 2559: RA, DEC, ZENITH DISTANCE PER SATELLITE

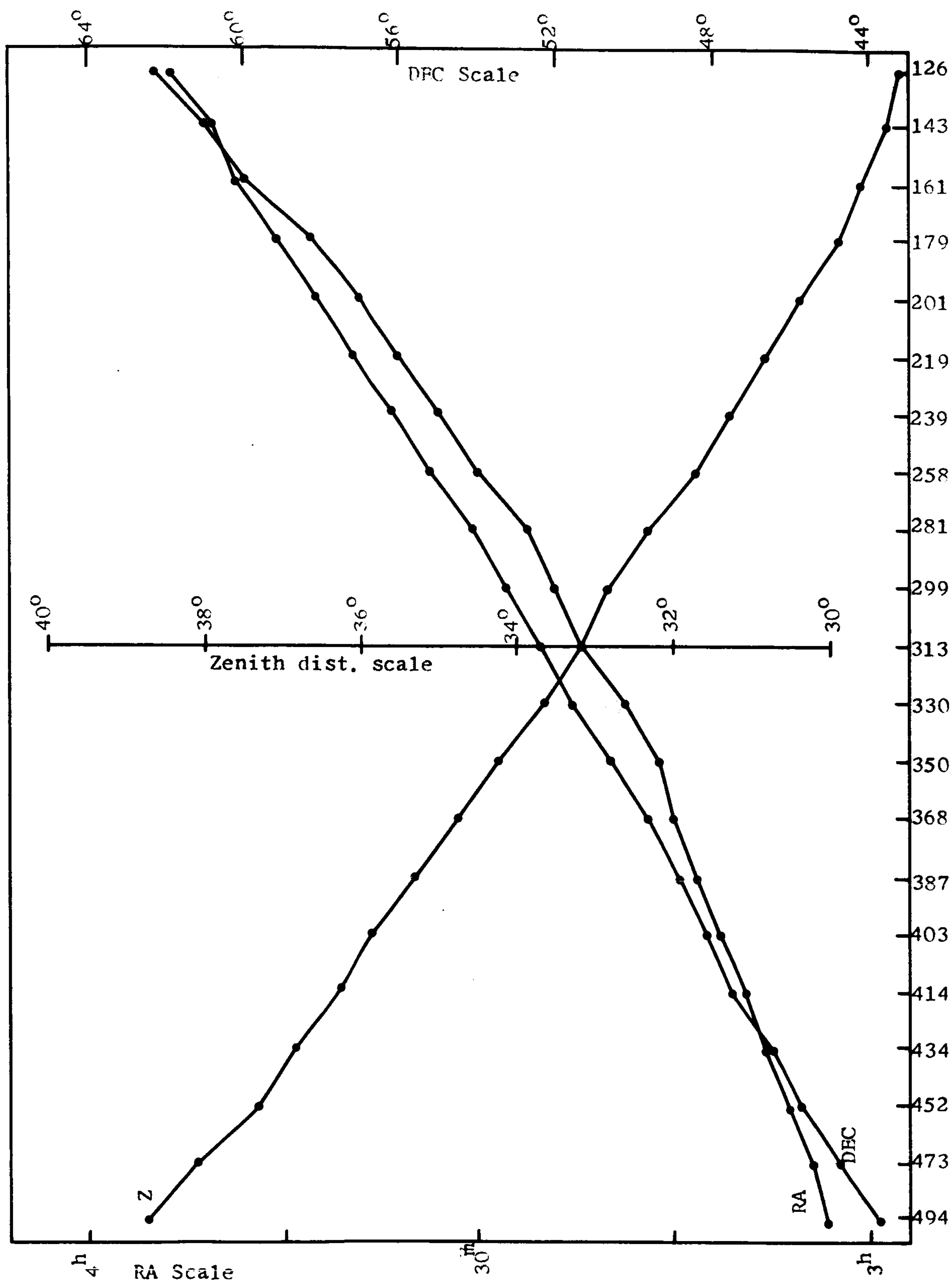

FIG. 8. PLATE 5205: RA, DEC, ZENITH DISTANCE PER SATELLITE

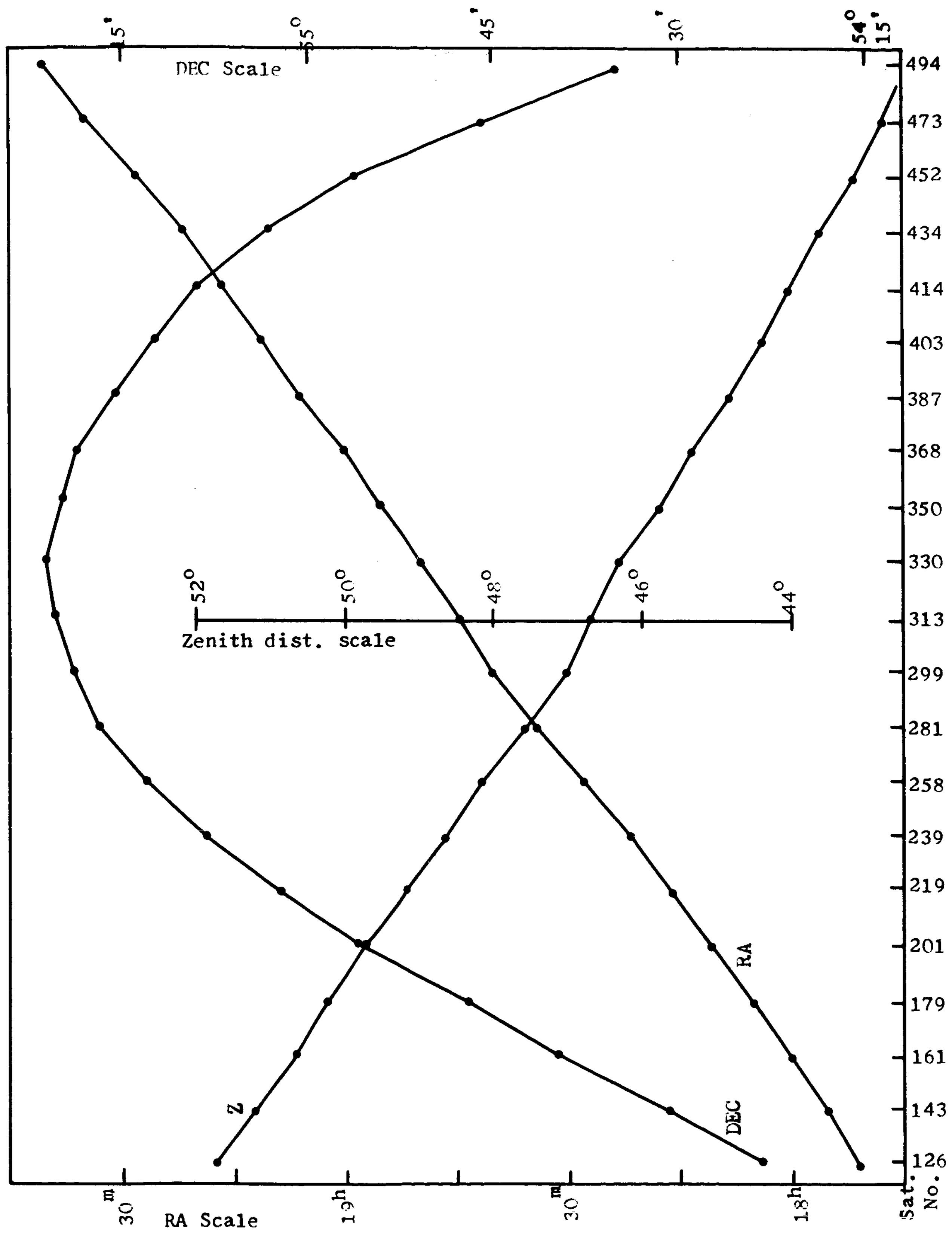

FIG. 9. PLATE 6132: RA, DEC, ZENITH DISTANCE PER SATELLITE

more than an expected amount from the other three agencies although one does not expect any agency to compare exactly with another. OSU had difficulty in finding an explanation for the differences. When ACIC was informed about the discrepancy, they rereduced the plates using new camera calibration parameters obtained from Duane Brown Associates who computed these from the data originally sent to NASA. The results are shown by dashed lines in the diagrams, and they indicate good agreement with the curves of the other agencies. The conclusion drawn was that the camera calibration program of ACIC must be in error or it is applied erroneously. Investigations in this regard lead to the suspicion that the program in question does not contain corrections for refraction; thus it should be used with zenith plates only. Since none of the ESSA-supplied plates were such, the program produced erroneous calibration parameters. It is not known to OSU whether or not PC-1000 data in the Data Center is also affected. No statement to the contrary was received from ACIC to date, though a request to clear up the matter was submitted in early May.

2.4 Short-Arc Orbit Constraint Adjustment of Satellite Observations and Comparison with Strictly Geometric Adjustment

The principle capabilities of the short-arc computer program were described in the First Semiannual Status Report. During this report period several minor improvements have been added to the program. These include (1) improvement of several printed output formats with the inclusion of more information, (2) grouping and analysis of residuals by orbit and by station, and (3) the capability of imposing chord distance constraints and relative position constraints among stations.

Several experiments have been performed involving the adjustment of a small amount of BC-4 data in the short-arc mode. The purpose of these experiments was to see if the imposition of the orbital constraint would have the same effect as the fitting of polynomials to the images on each plate. It has been the practice of the Coast & Geodetic Survey to fit 5th- or 7th-degree polynomials to each satellite trail. Since two polynomials are fit to each plate, and each event involves at least two plates, there are at least 24 independent unknowns introduced for each event by the curve fitting. Since the true path of the satellite is described by only 6 parameters, it appeared that it should be possible to describe the path of the satellite across all plates in the event with these 6 parameters only.

The data used for these experiments was as follows:

	Station
6002	Beltsville, Maryland
6003	Moses Lake, Washington
6038	Revilla Gigedo Island, Mexico

Event	Plates
2497	A404, B328
3539	A491, E393
4212	A567, U411
4236	A571, E422, U416
4267	A574, E450, U423

All images on each of these plates were corrected for parallactic refraction, phase angle, and parallactic abberation. Approximately 20 individual images, depending on the length of the trail, were selected from each plate. So that the same observations could be used in both the geometric and orbital modes, the individual images were made simultaneous by second-order interpolation. In all experiments the coordinates of Beltsville on the North American Datum were held fixed and the distance between Beltsville and Moses Lake was constrained. Plate U416 of event 4236 gave trouble and was deleted from the experiments early in the process.

The experiments with the geometric mode were:

(1) 3 individual observations per plate

(2) 10 individual observations per plate

(3) 20 individual observations per plate

(4) 3 fictitious images per plate, obtained by breaking each satellite trail into 3 segments, fitting an independent 3rd-degree polynomial to each segment, and taking a single observation from each curve fit

(5) 1 fictitious image per plate, using a 5th-degree polynomial curve fit for the entire trail, in the manner of the C&GS

The experiments using the short-arc program were:

(6) every 20th image, uncorrected for parallactic refraction, phase angle, or light-time. These observations were taken directly from the NSSDC tape

(7) 20 individual images per plate (corrected)

The results of these experiments are shown in Table 7. The corrections to the assumed coordinates are shown in meters with their standard deviations (reduced to $\sigma_0 = 1$) underneath. The results of the first four experiments were about as expected. The fifth experiment, using 1 fictitious image per plate, gave unbelievably low uncertainties. This can only be explained by the fact that the adjustment had only 2 degrees of freedom.

The results of experiment 3 in the geometric mode and experiment 7 in the orbital mode are strikingly similar. These two experiments used exactly the same data.

None of the geometric mode adjustments required iteration. Three iterations were required in the orbital mode before the σ_0 remained stable. The orbital mode adjustment showed several signs of being unstable, and many of the matrices involved appeared to be very ill-conditioned. This appears to be because the satellite trails were too short to afford a good determination of the orbit. The conditioning of these matrices and the question of how long an arc is needed will be investigated during the next report period.

Table 7

Experiment	Moses Lake x	Moses Lake y	Moses Lake z	Revilla Gigedo x	Revilla Gigedo y	Revilla Gigedo z	Degrees of Freedom	A priori Std Dev of Obs	σ_o	Std Dev of 1 Direction
1	-0.9	- 3.2	0.5	42.2	58.2	49.1	16	1"7	1.24	2"11
	(9.3	43.8	34.0)	(23.4	47.8	63.6)				
2	-5.9	-22.4	6.3	7.5	7.5	18.4	65	1.7	1.21	2.05
	(5.1	24.0	18.5)	(12.8	26.1	34.6)				
3	-2.4	5.4	-20.3	21.2	36.9	-29.9	148	1.7	1.10	1.87
	(3.4	15.7	12.4)	(8.7	17.5	23.1)				
4	0.5	11.1	-15.1	18.7	13.7	- 2.7	16	1.0	0.6	0.6
	(5.5	25.8	20.2)	(13.8	28.3	37.7)				
5	-1.8	5.7	-17.6	12.8	-7.6	26.6	2	0.7	0.3	0.21
	(6.7	32.0	24.7)	(16.7	34.7	46.0)				
6	1.3	34.1	-39.6	11.4	25.2	-15.1	290	1.7	0.9	1.5
	(5.0	21.1	15.5)	(13.1	24.1	29.9)				
7	-2.7	0.6	-14.3	19.8	30.8	-22.1	451	1.7	1.00	1.7
	(3.1	13.9	11.8)	(10.4	17.6	23.5)				

During the next report period, these experiments will be extended to a larger set of data involving 4 stations, 21 events, and 45 plates.

2.5 Sequential Least Squares Adjustment of Satellite Triangulation and Trilateration in Combination with Terrestrial Data

By means of satellite triangulation and trilateration, the coordinates of earth-fixed points are obtainable in a common coordinate system. This coordinate system is (a) "absolutely" or (b) "relatively" oriented. The former (a) is termed the average "terrestrial" coordinate system. It has as its origin the earth's center of gravity and is oriented so that the tertiary axis is directed toward the average pole as defined by the International Latitude Service; the primary-tertiary plane is parallel to the mean meridian of Greenwich as defined by the Bureau Internationale de l'Heure. The latter system (b) is termed the "geodetic" coordinate system and will be assumed to have axes which are parallel to the terrestrial system, i.e., (b) differs from (a) by a translation only.

In satellite triangulation absolute satellite directions, i.e., topocentric right ascensions and declinations, are simultaneously observed from several ground stations. The adjustment of these observations in a given network requires that one ground station be held fixed and the scale be obtained say from terrestrial distances and introduced via a terrestrial chord between any two ground stations. It is clear that the orientation of the network is determined via the absolute satellite directions implicit in the topocentric right ascensions and declinations.

On the other hand, satellite trilateration involves the observation of topocentric ranges from several ground stations. The adjustment of these observations requires that one ground station be held fixed and the orientation be accomplished by constraining two absolute terrestrial directions within the network. These absolute terrestrial directions may be in the form of direction numbers resulting from many absolute satellite directions simultaneously observed from pairs of ground stations. Still another form of the directions may be absolute three-dimensional geodetic normal section azimuths and altitudes. Also, the absolute terrestrial directions may be determined from terrestrial vertical angle and astronomic azimuth observations. Scale is implicit in the ranges themselves; however, scale may also be introduced by terrestrial bases as mentioned previously in the discussion concerning satellite triangulation.

The modern approach is to combine absolute satellite directions, ranges, and terrestrial data together in one adjustment. This system will be referred to as satellite triangulation and trilateration in combination with terrestrial data. In addition to absolute terrestrial directions and terrestrial spatial distances (observables already mentioned), the adjustment system to be developed will be shown to be capable of processing a mixture of satellite directions and ranges, astronomic latitude, longitude, azimuth, and vertical angle, deflection of the vertical components, geopotential numbers and geoid undulations or height anomalies and ground station coordinates.

In order to determine the effect that new observations and/or additional constraints may have on an original adjustment, a sequential adjustment system is developed. The following two specific sequential developments are given: Sequential modification of an original satellite triangulation adjustment by additional absolute satellite directions, by ranges, by a mixture of the previous two, and by terrestrial data; sequential modification of an original satellite trilateration adjustment by additional ranges, by absolute satellite directions, by a mixture of the previous two, and terrestrial data. The problems of the combination of normal equations, and the sequential combination of solutions is also treated.

Consideration is given to the design of the computerized sequential least squares adjustment system. Finally, the least squares sequential expressions are applied. Two three-segment computer programs have been developed to process terrestrial spatial distances and satellite range events, respectively. The computer programs along with some test runs will be reported during the next period (OSU Department of Geodetic Science Report No. 114).

3. PERSONNEL

Ivan I. Mueller, Project Supervisor, part time

Edward J. Krakiwsky, Research Associate, part time

Georges Blaha, Research Associate, part time

James P. Reilly, Research Assistant, part time

Charles R. Schwarz, Research Assistant, part time

Daniel Hornbarger, Research Assistant, without compensation

James Veach, Research Assistant, without compensation

Joseph E. Gross, III, Research Assistant, without compensation

Jeanne C. Preston, Research Aide, part time

Irene B. Tesfai, Technical Assistant, full time

John R. Miller, Research Aide, part time

Claire Lenfest, Technical Assistant, part time

Christine L. Jaeger, Technical Assistant, part time

Jane B. Sims, Technical Assistant, part time

William H. Wright, Research Aide, part time

4. TRAVEL

Trips made by project personnel during the report period are:

Ivan I. Mueller, Boulder, Colorado, February 27 - March 3, 1968
to attend Time and Frequency Seminar at the National Bureau
of Standards

Ivan I. Mueller, Washington, D.C. and Tampa,Florida, March 14-21, 1968
to attend meeting of the American Society of Photogrammetry
to attend conference on Photographic Astrometry

Daniel H. Hornbarger, Tampa, Florida, March 17-19, 1968
to attend conference on Photographic Astrometry

Ivan I. Mueller, Wallops Island, Virginia, June 5-6, 1968
to attend meeting of the NGSP Principal Investigators

5. REPORTS PUBLISHED TO DATE

OSU Department of Geodetic Science Reports published under Grant No. NSR 36-008-003:

70 The Determination and Distribution of Precise Time
by Hans D. Preuss
April, 1966

71 Proposed Optical Network for the National Geodetic Satellite Program
by Ivan I. Mueller
May, 1966

82 Preprocessing Optical Satellite Observations
by Frank D. Hotter
April, 1967

86 Least Squares Adjustment of Satellite Observations for Simultaneous Directions or Ranges, Part 1 of 3: Formulation of Equations
by Edward J. Krakiwsky and Allen J. Pope
September, 1967

87 Least Squares Adjustment of Satellite Observations for Simultaneous Directions or Ranges, Part 2 of 3: Computer Programs
by Edward J. Krakiwsky, George Blaha, Jack M. Ferrier
in press

88 Least Squares Adjustment of Satellite Observations for Simultaneous Directions or Ranges, Part 3 of 3: Subroutines
by Edward J. Krakiwsky, Jack Ferrier, James P. Reilly
December, 1967

93 Data Analysis in Connection with the National Geodetic Satellite Program
by Ivan I. Mueller
November, 1967

OSU Department of Geodetic Science Reports published under Grant No. NGR 36-008-093:

100 Preprocessing Electronic Satellite Observations
by Joseph Gross
March, 1968

106 Comparison of Astrometric and Photogrammetric Plate Reduction Techniques for a Wild BC-4 Camera
by Daniel H. Hornbarger
March, 1968

110 Investigations into the Utilization of Passive Satellite Observational Data
by James P. Veach
June, 1968

114 Sequential Least Squares Adjustment of Satellite Triangulation and
Trilateration in Combination with Terrestrial Data
by Edward J. Krakiwsky
in press